Math 130 Independent Project Topic Suggestions

1. Design a project of your own choice! E-mail me with what you propose to do. If you want help designing the project, or finding resources, talk to me in office hours.

2. Perform a critical examination of Book I of Euclids Elements. Discuss the proofs (and failings or short-comings in Euclids proofs) of a few propositions of your choice, ideally with a common theme. Discuss what additional justification, axioms, etc. would be necessary to make the proofs rigorous. Suggested first reading: online version of Euclid and commentary at `http://aleph0.clarku.edu/~djoyce/elements/bookI/bookI.html`.

3. Proof the only-if direction of Gauss' regular constructible n-gons. See the book *Conjecture & Proof* by M. Laczkovich, chapter 3. Alternatively, if you have experience with Galois theory, you can look at Dummit and Foote chapter 14, Prop. 14.29 for a proof of the other direction.

4. Prove that you cannot equidecompose a tetrahedron into a cube of the same volume. See the book *Conjecture & Proof* by M. Laczkovich, or section 27 of Hartshorne (these give different proofs, both are nice).

5. Explore constructions with compass and *marked* ruler (you have already trisected the angle this way). Show how to construct the regular 7-gon. What else can be done? Reference: Hartshorne chapter 30 and/or 31.

6. Explore constructions with origami. Show how to trisect an angle this way. What else can be done? Reference: `www.paperfolding.com` and `http://mars.wne.edu/~thull/origamimath.html`.

7. The Banach–Tarski paradox. Explain how to decompose sphere, rotate and translate the pieces, and produce two spheres (or a least get as far as the concept of paradoxical decompositions of sets). Requires some knowledge of group theory.

8. The "Cartesian plan" with coordinates in fields other than $\mathbb{R}$. You encountered $\mathbb{Q}^2$ in a homework question. Learn what happens for other fields. Can you use finite fields? Possible reference: Hartshorne chapters 14–15 (although there are many other resources).

9. What are the axioms for an *affine plane*? What are some interesting models for this geometry? Possible reference: part II of E. Moorehouse's book, available here: `http://www.uwyo.edu/moorhouse/handouts/incidence_geometry.pdf`.

10. Explore the notion of *Steiner system*. How does this relate to finite projective planes and finite affine planes (see the above). What can you say about the game *SET*?

11. Any other chapter from Hartshorne that we didn't cover in class. A few suggestions:
    - Euclid and Hilbert on area, sections 22–23.
    - Geometries based on algebra, sections 14–15.
    - Euclid's theory of volume, section 26.

12. Cover a topic from Stillwell's book that we didn't do in class (*e.g.* from chapter 6). Check with me first!

13. Learn how to construct an *error correcting code* using projective geometry. A quick search gives `http://micsymposium.org/mics_2009_proceedings/mics2009_submission_52.pdf` as a reference, but I'm sure you can find more!

14. There are 17 ways of tiling the Euclidean plan with repeating patterns (see `https://en.wikipedia.org/wiki/Wallpaper_group` for pictures). Learn why! This topic uses elementary group theory.

15. Explore the notion of *polytopes*, higher dimensional polygons. Can you determine all the regular polytopes? (Try not to get caught up in all the irregular/compound polytopes.) The class reference is *Regular Polytopes* by H. S. M. Coxeter, but I'm sure you can find less dense references online.